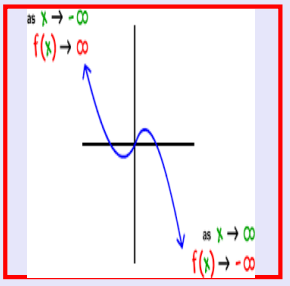
Math 245

Spring 2022

Lecture 30



Review

Divide using long division: $\dfrac{3x^3 - 5x^2 + 7x - 2}{x - 2}$

$$x - 2 \;\overline{)\; 3x^3 - 5x^2 + 7x - 2}$$

$3x^2 + x + 9$

$-(3x^3 - 6x^2)$

$x^2 + 7x - 2$

$-(x^2 - 2x)$

$9x - 2$

$-(9x - 18)$

Remainder $\longrightarrow 16$

$x \boxed{3x^2} = 3x^3$

$x \boxed{x} = x^2$

$x \boxed{9} = 9x$

$$\frac{3x^3 - 5x^2 + 7x - 2}{x - 2} = 3x^2 + x + 9 \quad R16$$

$$\frac{3x^3 - 5x^2 + 7x - 2}{x - 2} = 3x^2 + x + 9 + \frac{16}{x - 2}$$

Remainder

Always

Divide using Synthetic division: $\dfrac{3x^3 - 5x^2 + 7x - 2}{x - 2}$

$x - c \Rightarrow c = 2$

	x^3	x^2	x	Const.
2	3	−5	7	−2
		6	2	18
	3	1	9	16
	x^2	x	Const.	↑ Remainder

$3x^2 + x + 9 + \dfrac{16}{x-2}$

Divide using synthetic Division: $\dfrac{5x^3 - 7x + 8}{x + 4}$ (x^2 Missing)

$x - (-4)$

$5x^3 + 0x^2 - 7x + 8$

$x - (-4)$

$c = -4$

	x^3	x^2	x	
−4	5	0	−7	8
		−20	80	−292
	5	−20	73	−284
				↑ Remainder

$5x^2 - 20x + 73 + \dfrac{-284}{x+4}$

Always

$9 \div 4 = 2$ R 1 — Remainder

$\frac{9}{4} = 2$ R1

$\frac{9}{4} = 2 + \frac{1}{4}$

Dividend, Divisor, Quotient

$\frac{9}{4} = 2 + \frac{1}{4}$

Let's multiply both sides by divisor=4

$9 = 2 \cdot 4 + \frac{1}{4} \cdot 4 \Rightarrow 9 = 2 \cdot 4 + 1$

Dividend, Quotient, Divisor, Remainder

$\frac{P(x)}{D(x)} = Q(x)$ Remainder $r(x)$

$\frac{P(x)}{D(x)} = Q(x) + \frac{r(x)}{D(x)}$

multiply by $D(x)$

$P(x) = Q(x) \cdot D(x) + r(x)$

Dividend, Quotient, Divisor, Remainder

Divide $2x^2 - 5x - 1$ by $x - 3$

$2x^2 - 5x - 1$ — dividend; $x - 3$ — Divisor

$x - 3$

$x - c \Rightarrow c = 3$

3	2	−5	−1
		6	3
	2	1	2

Remainder

$\frac{2x^2 - 5x - 1}{x - 3} = 2x + 1 + \frac{2}{x - 3}$

Quotient

$2x^2 - 5x - 1 = (x - 3)(2x + 1) + 2$

Dividend, Divisor, Quotient, Remainder

$P(x) = D(x) \cdot Q(x) + r(x)$

Divide $\dfrac{3x^4 - 5x^3 + 7x - 8}{x+2}$

x^2 is missing

$x - (-2) \Rightarrow c = -2$

-2	3	-5	0	7	-8
		-6	22	-44	74
	3	-11	22	-37	66

$$\frac{3x^4 - 5x^3 + 7x - 8}{x+2} = 3x^3 - 11x^2 + 22x - 37 + \frac{66}{x+2}$$

$$3x^4 - 5x^3 + 7x - 8 = (x+2)(3x^3 - 11x^2 + 22x - 37) + 66$$

$$\frac{f(x)}{x-k} = q(x) + \frac{r}{x-k}$$

Multiply by $x - k$

$$f(x) = (x-k) \cdot q(x) + r$$

If $x = k$, then

$$f(k) = \underbrace{(k-k)}_{0} \cdot q(k) + r$$

0

So $f(k) = r$ (Remainder)

If the polynomial $f(x)$ is divided by $x-k$, then the remainder is $f(k)$.

Remainder Theorem

Suppose $f(x) = 2x^3 - 5x^2 + 3x - 7$

1) Find $f(2)$.

$f(2) = 2(2)^3 - 5(2)^2 + 3(2) - 7$

$= 2 \cdot 8 - 5 \cdot 4 + 3 \cdot 2 - 7$

$= 16 - 20 + 6 - 7 = -4 + 6 - 7 = 2 - 7 = \boxed{-5}$

2) Divide $f(x)$ by $x - 2$. $\Rightarrow K = 2$

$x - K$

2	2	-5	3	-7
		4	-2	2
	2	-1	1	-5 ← Remainder

Given $f(x) = 5x^3 - 6x^2 - 28x - 2$

1) Evaluate $f(-2)$

$f(-2) = 5(-2)^3 - 6(-2)^2 - 28(-2) - 2$

$= 5 \cdot (-8) - 6 \cdot 4 + 28 \cdot 2 - 2 =$

$= -40 - 24 + 56 - 2$

$= \boxed{-10}$

2) use Synthetic Division to divide $f(x)$ by $x + 2$.

$x - (-2)$

-2	5	-6	-28	-2
		-10	32	-8
	5	-16	4	-10